SADLER UNIT 3. CHAPTER 1

EXERCISE 1A

Q1. a) $\sqrt{-64} = \sqrt{64}\sqrt{-1}$
$= \underline{\underline{8i}}$

b) $\sqrt{-8} = \sqrt{8}\sqrt{-1}$
$= \underline{\underline{2\sqrt{2}i}}$

c) $\sqrt{-10} = \sqrt{10}\sqrt{-1}$
$= \underline{\underline{\sqrt{10}\,i}}$

d) $\sqrt{-63} = \sqrt{63}\sqrt{-1}$
$= \underline{\underline{3\sqrt{7}\,i}}$

Q2 a) $Re(z) = -5$
b) $Im(z) = 3$

Q3 a) $Re(z) = 12$
b) $Im(z) = -5$

Q4 a) $x = \dfrac{-(-3) \pm \sqrt{9-4(1)(3)}}{2(1)}$
$= \dfrac{3 \pm \sqrt{-3}}{2}$
$= \underline{\underline{\frac{3}{2} \pm \frac{\sqrt{3}}{2}i}}$

b) $x = \dfrac{-4 \pm \sqrt{16-4(1)(7)}}{2(1)}$
$= \dfrac{-4 \pm \sqrt{-12}}{2}$
$= \underline{\underline{-2 \pm \sqrt{3}i}}$

c) $x = \dfrac{-(-1) \pm \sqrt{1-4(3)(1)}}{2(3)}$
$= \dfrac{1 \pm \sqrt{-11}}{6}$
$= \underline{\underline{\frac{1}{6} \pm \frac{\sqrt{11}}{6}i}}$

d) $x = \dfrac{-8 \pm \sqrt{64-4(5)(4)}}{2(5)}$
$= \dfrac{-8 \pm \sqrt{-16}}{10}$
$= -\frac{4}{5} \pm \frac{2}{5}i$ //

Q5. $(3+7i) + (2-i)$
$= 3+2+7i-i$
$= \underline{\underline{5+6i}}$

Q6. $(1-2i) - (3-2i)$
$= 1-3-2i+2i$
$= \underline{\underline{-2}}$

Q7. $12+4i-2-5i$
$= \underline{\underline{10-i}}$

Q8. $6-i+3+4i$
$= \underline{\underline{9+3i}}$

Q9. $(1+i)+(3-2i)+(4-i)$
$= 1+3+4+i-2i-i$
$= \underline{\underline{8-2i}}$

Q10. $2(5-2i)+2(-5+3i)$
$= 10-4i-10+6i$
$= \underline{\underline{2i}}$

Q11. $7(1-3i)+15i$
$= 7-21i+15i$
$= \underline{\underline{7-6i}}$

Q12. $5+3(4+2i)$
$= 5+12+6i$
$= \underline{\underline{17+6i}}$

Q13. $Re(5+2i) + Re(-3+4i)$
$= 5-3$
$= \underline{\underline{2}}$

Q14. $Im(-1-7i) + Im(3+2i)$
$= -7+2$
$= \underline{\underline{-5}}$

Q15 $(5-2i)(2+3i)$
$= 10+15i-4i-6i^2$
$= 10+6+11i$
$= \underline{\underline{16+11i}}$

Q16 $(3+i)(3+2i)$
$= 9+6i+3i+2i^2$
$= 9-2+9i$
$= \underline{\underline{7+9i}}$

Q17. $(2+i)(2-i)$
$= 4-i^2$
$= 4+1$
$= \underline{\underline{5}}$

Q18 $(-2+7i)(7-2i)$
$= -14+4i+49i-14i^2$
$= -14+14+53i$
$= \underline{\underline{53i}}$

Q19. $\frac{2-3i}{1+2i} \times \frac{1-2i}{1-2i}$
$= \frac{(2-3i)(1-2i)}{1+4}$
$= \frac{2-4i-3i+6i^2}{5}$
$= \frac{-4-7i}{5}$
$= \underline{\underline{-\frac{4}{5}-\frac{7}{5}i}}$

Q20. $\frac{2-3i}{2+3i} \times \frac{2-3i}{2-3i}$
$= \frac{(2-3i)^2}{4+9}$
$= \frac{4-12i+9i^2}{13}$
$= \frac{-5-12i}{13}$
$= -\frac{5}{13}-\frac{12}{13}i$ //.

Q21. $\frac{5-2i}{3+4i} \times \frac{3-4i}{3-4i}$
$= \frac{(5-2i)(3-4i)}{9+16}$
$= \frac{15-20i-6i+8i^2}{25}$
$= \frac{7-26i}{25}$
$= \underline{\underline{\frac{7}{25}-\frac{26}{25}i}}$

Q22 $\frac{i}{2-i} \times \frac{2+i}{2+i}$
$= \frac{i(2+i)}{4+1}$
$= \frac{2i+i^2}{5}$
$= \underline{\underline{-\frac{1}{5}+\frac{2}{5}i}}$

Q23. $\boxed{w=2+3i}$ $\boxed{z=5-i}$

a) $w+z = 2+3i+5-i$
$= \underline{\underline{7+2i}}$

b) $w-z = 2+3i-(5-i)$
$= 2-5+3i+i$
$= \underline{\underline{-3+4i}}$

c) $5w-4z = 5(2+3i)-4(5-i)$
$= 10+15i-20+4i$
$= \underline{\underline{-10+19i}}$

d) $wz = (2+3i)(5-i)$
$= 10-2i+15i-3i^2$
$= \underline{\underline{13+13i}}$

e) $z^2 = (5-i)^2$
$= 25-10i+i^2$
$= \underline{\underline{24-10i}}$

f) $\frac{w}{z} = \frac{2+3i}{5-i} \times \frac{5+i}{5+i}$
$= \frac{10+2i+15i+3i^2}{25+1} = \underline{\underline{\frac{7}{26}+\frac{17}{26}i}}$

Q24 $\boxed{z = 4 - 7i}$

a) $\bar{z} = 4 + 7i$

b) $z + \bar{z} = 4 - 7i + 4 + 7i$

$= \underline{\underline{8}}$

c) $z\bar{z} = (4 - 7i)(4 + 7i)$

$= 16 - 49i^2$

$= 16 + 49$

$= \underline{65}$

d) $\dfrac{z}{\bar{z}} = \dfrac{4 - 7i}{4 + 7i} \times \dfrac{4 - 7i}{4 - 7i}$

$= \dfrac{16 - 56i + 49i^2}{16 + 49}$

$= \dfrac{-33 - 56i}{65}$

$= \underline{\underline{-\dfrac{33}{65} - \dfrac{56}{65} i}}$

Q25 $\boxed{z = 5 + ai}$ $\boxed{w = b - 34i}$

$5 + ai = b - 34i$

$\Rightarrow \underline{\underline{b = 5}} \quad \underline{\underline{a = -34}}$

Q26 $(a + 5i)(2 - i) = b$

$2a - ai + 10i - 5i^2 = b$

$(2a + 5) + (10 - a)i = b$

$2a + 5 = b$

$10 - a = 0$

$\boxed{a = 10}$

$\therefore 2(10) + 5 = b$

$\boxed{b = 25}$

Q27. $ax^2 + bx + c = 0$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

a)

If $b^2 - 4ac = -k$, $k \in \mathbb{R}^+$,

then $x = -\dfrac{b}{2a} \pm \dfrac{\sqrt{-k}}{2a}$

$= -\dfrac{b}{2a} \pm \dfrac{\sqrt{k}}{2a} i$

$\therefore$ 2 complex conjugate roots.

b) METHOD 1: If $x = 2 + 3i$, then

$\bar{x} = 2 - 3i$.

$(x - 2 - 3i)(x - 2 + 3i)$

$= x^2 - 2x + \cancel{3xi} - 2x + 4 - \cancel{6i} - \cancel{3xi} + \cancel{6i} - 9i^2$

$= x^2 - 4x + 4 + 9$

$= \underline{\underline{x^2 - 4x + 13}}$

$\Rightarrow p = -4$ and $q = 13$.

METHOD 2:

$(2 + 3i)^2 + p(2 + 3i) + q = 0$

$4 + 12i + 9i^2 + 2p + 3pi + q = 0$

$(4 - 9 + 2p + q) + (12 + 3p)i = 0$

$\Rightarrow 12 + 3p = 0$

$3p = -12$

$\underline{\underline{p = -4}}$

$-5 + 2(-4) + q = 0$

$-13 + q = 0$

$\underline{\underline{q = 13}}$

c) METHOD 1: If $x = 3 - 2i$, then

$\bar{x} = 3 + 2i$

$(x - 3 + 2i)(x - 3 - 2i)$

$= x^2 - 3x - \cancel{2xi} - 3x + 9 + \cancel{6i} + \cancel{2xi} - \cancel{6i} - 4i^2$

$= x^2 - 6x + 9 + 4$

$= \underline{\underline{x^2 - 6x + 13}}$

$\Rightarrow d = -6$ and $e = 13$

METHOD 2

$(3 - 2i)^2 + d(3 - 2i) + e = 0$

$9 - 12i + 4i^2 + 3d - 2di + e = 0$

$(9 - 4 + 3d + e) - (12 + 2d)i = 0$

$12 + 2d = 0$

$2d = -12$

$\underline{\underline{d = -6}}$

$5 + 3(-6) + e = 0$

$-13 = -e$

$\underline{\underline{e = 13}}$

Q28.

a) $(5,1)+(-3,2)$

$= \underline{\underline{(2,3)}}$

b) $(-2,3)-(1,3)$

$= \underline{\underline{(-3,0)}}$

c) $(2,0)\times(2,1)$

$= 2(2+i)$

$= 4+2i$

$= \underline{\underline{(4,2)}}$

d) $(5,-1)\div(-5,12)$

$= \dfrac{5-i}{-5+12i} \times \dfrac{-5-12i}{-5-12i}$

$= \dfrac{(5-i)(-5-12i)}{25+144}$

$= \dfrac{-25-60i+5i+12i^2}{169}$

$= \dfrac{-25-12-55i}{169}$

$= \underline{\underline{-\dfrac{37}{169} - \dfrac{55}{169}i}}$

Q29. $\dfrac{14-5i}{a-4i} = 2+bi$

$14-5i = (a-4i)(2+bi)$

$= 2a+abi-8i-4bi^2$

$= (2a+4b)+(ab-8)i$

$ab-8=-5$

$ab=3$

$\underline{\underline{b=\dfrac{3}{a}}}$

$2a+4\left(\dfrac{3}{a}\right)=14$

$2a^2+12=14a$

$2a^2-14a+12=0$

$a^2-7a+6=0$

$(a-1)(a-6)=0$

$\therefore \underline{\underline{(a,b)=(1,3)}}$ or $\underline{\underline{\left(6,\tfrac{1}{2}\right)}}$

EXERCISE 1B

Q1. Let $x=1$,

$2(1)^3+(1)^2+p(1)+35=0.$

$2+1+p+35=0$

$p+38=0$

$\underline{\underline{p=-38}}$

Q2. $x^3+3x^2-2x-16 \Rightarrow f(x)$

$f(1)=1+3-2-16\neq 0$

$f(2)=8+12-4-16=0$

$\therefore (x-2)$ is a factor

2	1	3	−2	−16
		2	10	16
	1	5	8	0

$\therefore (x-2)(x^2+5x+8)$

$\therefore \underline{\underline{a=2}}, \underline{\underline{b=1}}, \underline{\underline{c=5}}, \underline{\underline{d=8}}$

Q3 a) $\dfrac{x^2-7x+3}{x-1}$

$= \dfrac{x(x-1)-6(x-1)-3}{x-1}$

$= x-6-\dfrac{3}{x-1}$

$\therefore$ Remainder of $\underline{\underline{-3}}$.

b) $f(x)=x^2-7x+3$

$f(1)=1-7+3$

$=-6+3$

$=\underline{\underline{-3}}$

Q4a) $\dfrac{2x^3+3x^2-4x+3}{x+1}$

$= \dfrac{2x^2(x+1)+x(x+1)-5(x+1)+8}{x+1}$

$= 2x^2+x-5+\dfrac{8}{x+1}$

$\therefore$ Remainder of $\underline{\underline{8}}$.

b) $f(x)=2x^3+3x^2-4x+3$

$f(-1)=2(-1)+3(1)-4(-1)+3$

$=-2+3+4+3.$

$=\underline{\underline{8}}$

Q5. $x^2+3x-6 \Rightarrow f(x)$

$f(2)=4+6-6$

$=\underline{\underline{4}}$

Q6. $x^3-5x^2-8x+7 \Rightarrow f(x)$

$f(-2)=-8-5(4)-8(-2)+7$

$=-8-20+16+7$

$=\underline{\underline{-5}}$

Q7. $f(x)=2x^3+ax^2+bx-2$

$f(\tfrac{1}{2})=2(\tfrac{1}{8})+a(\tfrac{1}{4})+b(\tfrac{1}{2})-2$

$0=\tfrac{1}{4}+\tfrac{1}{4}a+\tfrac{1}{2}b-2$

$0=1+a+2b-2$

7 = a + 2b ①

$f(-1)=2(-1)+a(1)+b(-1)-2$

$-6=-2+a-b-2$

$-2=a-b$ ②

① − ② : *9 = 3b*

b = 3

$\therefore$ *a = 7 - 2(3)*

a = 1

Q8a) $f(x)=x^3-3x^2+7x-5$

$f(-1)=-1-3(1)+7(-1)-5$

$=-1-3-7-5$

$=\underline{\underline{-16}}$

$f(1)=1-3(1)+7(1)-5$

$=1-3+7-5$

$=\underline{\underline{0}}$

b) $\therefore (x-1)$ is a factor

1	1	−3	7	−5
		1	−2	5
	1	−2	5	0

$\therefore (x-1)(x^2-2x+5)=0$

$\underline{\underline{x=1}}$ or $x=\dfrac{-(-2)\pm\sqrt{4-4(1)(5)}}{2}$

$=\dfrac{2\pm\sqrt{-16}}{2}$

$=\underline{\underline{1\pm 2i}}$

c) $x(x^3-3x^2+7x-5)=0$

$\underline{\underline{x=0,\ 1,\ 1\pm 2i}}$

Q9a) $f(x)=x^4-5x^3-x^2+11x-30$

$f(-2)=16-5(-8)-4+11(-2)-30$

$=16+40-4-22-30$

$=\underline{0}$

$f(2)=16-5(8)-4+22-30$

$=16-40-4+22-30$

$=38-74$

$=\underline{\underline{-36}}$

$f(-5)=625-5(-125)-25-55-30$

$=\underline{1140}$

$f(5)=625-625-25+55-30$

$=0$ //

b) $(x-5)$ and $(x+2)$ are factors.

Need: $x^4-5x^3-x^2+11x-30$

$$\frac{x^2(x^2-3x-10)-2x(x^2-3x-10)+3(x^2-3x-10)}{x^2-3x-10}$$

$\therefore (x-5)(x+2)(x^2-2x+3)=0$

$x=5$, $x=-2$, $x=\frac{-(-2)\pm\sqrt{4-4(1)(3)}}{2}$

$=\frac{3\pm\sqrt{-8}}{2}$

$=\frac{3}{2}\pm\frac{2\sqrt{2}}{2}i$

$=\frac{3}{2}\pm\sqrt{2}i$

Q10a) $f(x)=2x^3-x^2+2x-1$

$f(1)=2-1+2-1$

$=2$

$f(\frac{1}{2})=2(\frac{1}{8})-(\frac{1}{4})+1-1$

$=0$

$\therefore (2x-1)$ is a factor.

b) $\frac{x^2(2x-1)+1(2x-1)}{(2x-1)}$

$=x^2+1$

$\therefore f(x)=(2x-1)(x^2+1)$

$0=(2x-1)(x^2+1)$

$x=\frac{1}{2}$ or $x^2=-1$

$x=\pm i$

Q11. $(x^2+2x+2)(x^2-2x+5)=0$

$(x+1)^2-1+2=0$ $\quad$ $(x-1)^2-1+5=0$

$(x+1)^2=-1$ $\quad$ $(x-1)^2=-4$

$x+1=\pm i$ $\quad$ $x-1=\pm 2i$

$x=-1\pm i$ $\quad$ $x=1\pm 2i$

Q12. $2x^3-3x^2+9x-8=0$

By the rational root theorem, $\exists \pm\frac{a}{b}$ s.t.

$a|-8$ and $b|2$

such that $hcf(a,b)=1$

$a: 1,-1,2,-2,4,-4,8,-8$

$b: 1,-1,2,-2$

$\frac{a}{b}=\{1,2,4,8,\frac{1}{2}\}$.

$f(1)=2-3+9-8$

$=-1+1$

$=0$

$$\begin{array}{r|rrrr} 1 & 2 & -3 & 9 & -8 \\ & & 2 & -1 & 8 \\ \hline & 2 & -1 & 8 & 0 \end{array}$$

$\therefore (x-1)(2x^2-x+8)=0$

$x=1$ or $x=\frac{-(-1)\pm\sqrt{1-4(2)(8)}}{2(2)}$

$=\frac{1\pm\sqrt{1-64}}{4}$

$=\frac{1}{4}\pm\frac{3\sqrt{7}}{4}i$

Q13.

$3x^4 - 3x^3 - 2x^2 + 4x = 0$

$x(3x^3 - 3x^2 - 2x + 4) = 0$

By rational root theorem, $\exists \pm \frac{a}{b}$ such that $a \mid 4$ and $b \mid 3$. and $hcf(a, b) = 1$.

$a: 1, -1, 2, -2, 4, -4$

$b: 1, -1, 3, -3$

$\therefore \frac{a}{b} = \{1, 2, 4, \frac{1}{3}\}$

$f(-1) = 3(-1) - 3(1) - 2(-1) + 4$

$= 0$.

$$\therefore \begin{array}{r|rrrr} -1 & 3 & -3 & -2 & 4 \\ & & -3 & 6 & -4 \\ \hline & 3 & -6 & 4 & 0 \end{array}$$

$\therefore x(x+1)(3x^2 - 6x + 4) = 0$

$x = 0$, $x = -1$

$$x = \frac{-(-6) \pm \sqrt{36 - 4(3)(4)}}{2(3)}$$

$$= \frac{6 \pm \sqrt{-12}}{6}$$

$$= 1 \pm \frac{2\sqrt{3}}{6} i$$

$$= 1 \pm \frac{\sqrt{3}}{3} i$$